ECE5550 Applied Kalman Filtering

Autonomous & Intelligent Systems Labratory

Jaxon Topel

**Notes 4 : Linear Kalman Filter**

Goal : Learn how to estimate the present state of a noisy dynamic system, using measurements related to the state vector.

Estimation Theory: Our approach to optimal estimation was derived from the expected length squared of the error vector. Also known as the minimum mean square error (MMSE).  
Framework: Our state estimate is the conditional mean.

* From deriving the equation, we can compute desired density with two steps per iteration: Predicting and updating.
* We will assume that all probability densities are gaussian.
* Assuming gaussian means that we consider all random variables involved (initial state, process noise, measurement noise) follow a normal distribution.
  + State Estimation: The state of the system is represented by a mean vector and a covariance matrix. The mean vector provides the best estimate of the state, while the covariance matrix describes the uncertainty of this estimate.
* Predict: Filter uses the model to predict the state at the next time step and update the covariance matrix to reflect the increased uncertainty due to process noise.
* Update: Filter incorporates new measurements, updating the state estimate and reducing uncertainty based on measurement noise.

Gaussian Assumption:

* Error is always computed as truth minus the prediction.
* We cant ever compute the error in practice since truth value is not known.
* At the end of each iteration we have computed our best guess of the present state.

6 Steps of any Kalman Filter:

* Step 1a: State Estimate Time update
* Step 1b: Error Covariance time update
* Step 1c: Estimate System output
* Step 2a: Estimator gain matrix
* Step 2b: State estimate measurement update.
* Step 2c: Error covariance measurement update.

Optimal Application to linear systems:

* If the system dynamics are linear, then the Kalman filter is the optimal MMSE and maximum likelihood estimator.
* Step 1a: State Estimate Time update
  + In this step due to assumption of gaussian distribution, random process noise is zero mean.
  + In this step we predict the present state given only past measurements
  + Use most recent state estimate and propagate forward in time.
* Step 1b: Error Covariance time update
  + First we compute estimation error, truth – prediction.
* Step 1c: Estimate System output
* Step 2a: Estimator gain matrix
  + We first need to compute several covariance matrices.
  + This calculation of L\_k is the part that distinguishes from other estimation methods.
  + The first component to L\_k indicates need for correction to xhat, and how well states within xhat are coupled.
  + A large entry in the first component means state is very uncertain.
  + Small means state is certain.
  + C T k term gives the coupling between state and output.
  + SigmaZ term tells us how certain we are that the measurement is reliable
    - Large we want small, slow updates.
    - Small we want big updates.
    - This is why we divide the Kalman filter by sigmaZ.
* Step 2b: State estimate measurement update.
* Step 2c: Error covariance measurement update.